1997 V25 3: pp. 453–485
REAL ESTATE
ECONOMICS

The Value of Federal Sponsorship: The Case of Freddie Mac

James F. Gatti* and Ronald W. Spahr**

The Federal Housing Enterprises Financial Safety and Soundness Act of 1992 attempts to limit the risk borne by the taxpayer due to federal sponsorship of the Federal Home Loan Mortgage Corporation (FHLMC) and the Federal National Mortgage Association by: (1) controlling their range of activities; (2) requiring a level of capital sufficient to absorb substantial losses; and (3) providing a mechanism for closure if capital is insufficient. This article estimates the impact of the capital standards on the value to the FHLMC of federal sponsorship. Although FHLMC's level of capital exceeds requirements, the federal government still bears a nontrivial portion of the FHLMC's risk.

Losses imposed on the taxpayer as a result of the financial collapse of the Farm Credit System and the Federal Savings and Loan Insurance Corporation (FSLIC) and the need to recapitalize the FDIC have increased the awareness of and concern over financial guarantees issued by the federal government. The Financial Institutions Reform, Recovery, and Enforcement Act of 1989 (FIRREA) required the Congressional Budget Office, the General Accounting Office (GAO), the Office of Management and Budget and the U.S. Treasury to evaluate the risk posed to the federal government and the taxpayer by the operations of Government Sponsored Enterprises (GSEs). While this did not affect, in any way, the operations of GSEs, it did serve to address the question of the magnitude of the potential problem.

The Federal Housing Enterprises Financial Safety and Soundness Act of 1992 (FHEFSSA) is the first, and so far the only, positive step taken to limit the risk imposed on the taxpayer by GSEs, in this case the Federal Home Loan Mortgage Corporation (FHLMC or Freddie Mac) and the Federal National Mortgage Association (FNMA or Fannie Mae). Both entities are central players in the secondary market for conventional mortgages. In 1993, their combined activities accounted for 54.0% of conventional residential mortgage financing. This was down from 64.0% in 1992 and up from 51.0% in 1991. Their relative shares for 1993 were 45.0% for Freddie Mac and

*University of Vermont, Burlington, VT 05405
**University of Wyoming, Laramie, WY 82071

55.0% for Fannie Mae. Freddie Mac's share has risen from 44.0% and 43.0% in 1992 and 1991. Given the implicit federal guarantees that accompany their status as GSEs, they have very little competition in the market for securitizing conventional mortgage loans that conform to their underwriting standards/limitations.[1]

This article develops a general methodology for assessing the magnitude of the risk imposed on the federal taxpayer by any GSE, applies that methodology to the FHLMC and examines the sensitivity of that risk to changes in Freddie Mac's capital base. The results indicate that: (1) the new control standards lower the level of risk; (2) the level of risk is modestly sensitive to the degree of capitalization; (3) complete privatization of Freddie Mac (and, by implication, Fannie Mae) will raise costs only slightly but possibly by enough to permit additional private sector competition; and (4) while the implicit guarantee of Freddie Mac results in rather small subsidies given normal markets for real estate, the losses to the federal government and, therefore, the value of the guarantee rises substantially in the event of a severe recession in which the market value of real estate falls appreciably.

A Primer on Freddie Mac

A government sponsored enterprise, as defined in the Omnibus Budget Reconciliation Act of 1990 (OBRA), is a private corporation that operates under a charter granted by the Congress of the United States. The preponderance of its board of directors is elected by private shareholders, though some portion may be appointed by Congress or the president. It's central function is to serve as a financial intermediary, making loans or issuing loan guarantees to borrowers or sectors identified in the enabling legislation as being inadequately served by private markets. Funds may be raised in a variety of ways, but in no case are the liabilities of the GSE *explicitly* backed by the full faith and credit of the federal government.[2] In spite of this disclaimer, every GSE is perceived by the credit markets as having an *implicit* federal government guarantee backing its obligations. In addition, GSEs benefit from one or more *explicit* regulatory preferences such as federal tax exemptions, exemptions from SEC regulations and access to the U.S. Treasury for a line of credit. Since GSEs generally compete directly

[1] *Information Statement*, Federal Home Loan Mortgage Corporation, March 31, 1994, pp. 22–23.

[2] *Omnibus Budget Reconciliation Act of 1990*, PL 101-508, November 5, 1990. This disclaimer is repeated in Title XIII of FHEFSSA and in financial and operating reports issued by Freddie Mac.

with privately owned firms where the implied credit guarantee and other exemptions amount to subsidies, the GSEs have a competitive advantage of uncertain magnitude over private competitors.

Chartered in 1970, Freddie Mac was established to enhance the availability of credit in the residential mortgage market. Initially capitalized by issuing nonvoting common stock purchased by the Federal Home Loan Banks (themselves GSEs), Freddie Mac became completely privately owned with the passage of FIRREA in 1989. All ownership ties with the Federal Home Loan Banks were severed in 1990 when the nonvoting stock was retired and new common stock issued to the public. At the same time its board of directors was restructured—thirteen of the eighteen directors are elected by common stockholders, the rest are appointed by the president of the U.S.[3]

In passing FIRREA, Congress maintained a direct relationship between Freddie Mac and the public sector by placing it under the supervision of the secretary of Housing and Urban Development. Freddie Mac also retained a U.S. Treasury line of credit of up to $2.25 billion. This linkage makes Freddie Mac and other GSEs unique among publicly traded corporations, and it is presumably sufficient to justify the continued assumption that their obligations are free of default risk.[4]

The essential function of Freddie Mac, like that of its sister, Fannie Mae and second cousin Ginnie Mae, is to increase the availability of credit in the market for residential mortgages by establishing and maintaining a secondary market for the instruments. Freddie Mac does this by purchasing mortgages from originators and financing their acquisition either by securitizing the mortgages (selling mortgage-backed securities (MBS)), or by issuing ordinary debt. In 1993, 88.9% of all mortgages financed by Freddie Mac were securitized as compared to 92.8% in 1992.[5] The MBS in its various forms represents an ownership interest in the underlying mortgages and the latter are treated, for accounting purposes, as having been sold. Consequently, neither the mortgages nor the MBSs used to finance them appear as assets or liabilities on the balance sheet of Freddie Mac.

---

[3] *Financial Institutions Reform, Recovery, and Enforcement Act*, Public Law 101-101-73—August 9, 1989, p. 431.

[4] See, for example, Hemel, 1990 and Caso, 1990.

[5] *Information Statement*, March 31, 1994, p. 9. The figure used to calculate the retained mortgage portfolio excludes those mortgages that back multiclass debt securities. These mortgages are not securitized, but the nature of the multiclass instrument used to finance them means that they impose no interest rate risk on the FHLMC.

This is, however, an accounting fiction. Freddie Mac guarantees timely payment of interest and principal on the underlying instruments, making the sale of mortgages conditional, ownership reverting to the FHLMC in the event of default by the ultimate borrower. The MBS is, in fact, a contingent liability and the FHLMC recognizes it as such.

Given the structure of its operations, the risk borne by Freddie Mac and, by implication, the federal government, has two sources—default or credit risk and interest rate risk, the former being, by far, the more important in the case of the FHLMC. The federal government is insulated from the credit risk by four factors: (1) the loan to value ratio (LTV) of the mortgages at the time of origination; (2) loan loss reserves; (3) equity capital; and (4) the geographic diversification of the mortgage portfolio. Freddie Mac will not purchase mortgages with LTVs above 80.0% without private mortgage insurance on the excess. Because of loan amortization and increasing real estate values, the average LTV of sold and retained mortgages was estimated to be 66.0% as of the end of 1993. Also, as of the end of 1993 loss reserves were $760 million, while book value of equity was $4,437 million. Freddie Mac estimates that on a mark-to-market basis, equity capital was $5.6 billion before taxes and $5.2 billion after the estimated tax liability.[6] The geographic distribution of mortgage purchases is roughly proportional to the distribution of population, a reasonable proxy for the geographic distribution of residential construction activity and the housing stock.

Freddie Mac bears almost no interest rate risk on its securitized financing, because interest payments on the MBS are tied to the interest paid on the underlying mortgages. Consequently, market values of the assets rise and fall by the same amount as market values of the contingent liabilities. What little interest-rate risk exists in the securitized part of the operation comes from two sources: (1) reinvestment of cash flows prior to disbursement; and (2) the forty-five day delay between the purchase of mortgages and the sale of PCs to finance them.[7]

The Federal Housing Enterprises Financial Safety and Soundness Act

The regulatory structure established by the FHEFSSA is quite similar to that for banking and thrift institutions. The act creates, within the Department of

[6] *Information Statement*, March 31, 1994, pp. 13–45 and A-34.

[7] For more detail on the institutional characteristics of Freddie Mac and other GSEs see Gatti, Spahr; *Information Statement*, 1993; Stanton, 1991; and United States Treasury, 1990.

Housing and Urban Development, the Office of Housing Enterprise Oversight (OFHEO) whose responsibility it is to see that the provisions of the act are enforced either directly or through development of such additional regulations as are deemed necessary to accomplish the purposes of the act. The act gives the secretary of HUD and the director of OFHEO very broad authority to regulate financial and operational policies of the housing GSEs. Capital distributions and executive compensation may be limited, and asset selection directed to achieve specific distributional purposes of the act. In order to monitor compliance, the act gives OFHEO broad authority to require financial and operational information, requires annual examinations and permits the director of OFHEO to order additional examinations as deemed necessary.

Regulations that attempt to limit risk are of three kinds: (1) restrictions on the range of activities in which Fannie Mae and Freddie Mac are permitted to engage; (2) a requirement that the capital base of each entity be large enough to absorb foreseeable operating and/or capital losses; and (3) a mechanism for assuming control of the GSE by means of a conservator if insufficient capital cannot be made available. The act also directs the comptroller general, HUD and congressional Budget Office to examine the impact of revoking the federal charters of Freddie and Fannie.[8] Presumably successfully severing federal ties would eliminate the need for the regulation established by the act.

The act provides that within eighteen months from the date the OFHEO director is appointed, risk-based capital standards must be in place. These standards are to be developed by applying a "stress test" to the GSE. The test requires that the level of total capital be sufficient to withstand, for a period of ten years, losses that are likely to be associated with extreme levels of default experience and interest rate variation.[9] The default rate is to be based on the highest rate experienced over at least a two year period by a contiguous geographic region of the U.S. that contains at least 5.0% of the U.S.. The formula for calculating the assumed interest rate change is linked

[8] Federal Housing Enterprises Financial Safety and Soundness Act of 1992, Title XIII of the *Housing and Community Development Act of 1992*, Public Law 102-550, Oct. 28, 1992, pp. 3970–3971.

[9] Total capital is defined as core capital (par or stated value of common and perpetual preferred stock, core capital plus paid-in capital plus retained earnings) plus allowance for foreclosure losses excluding reserves held against specific assets plus "...other amounts from sources of funds available to absorb losses...that the Director...determines...appropriate..." *Housing and Community Development Act of 1992*, pp. 3942 & 3944.

to the ten-year constant maturity Treasury yield. The interest rate level assumed to prevail at the start of the stress period is either "...the lesser of: (1) 600.0 basis points (b.p.) below the average yield during the preceding nine months; or (2) 60.0% of the average yield during the preceding three years, but in no case to a yield less than 50.0% percent of the average yield during the last nine months," or "...the greater of: (1) 600.0 b.p. above the average rate for the preceding nine months; or (2) 160.0% of the average yield during the preceding three years, but in no case to a yield greater than 175.0% of the average yield during the preceding nine months."[10] As part of the total capital required to meet risk-based standards, the housing GSEs must hold core capital equal to minimum capital requirements of 250.0 b.p. of on-balance sheet assets and 45.0 b.p. of the unpaid balance of securitized mortgages.

If the institution meets both the risk-based and minimum levels, it will be classified as adequately capitalized. It will be considered undercapitalized if it fails to meet the risk-based standard but does succeed in meeting the minimum standard. Significant under-capitalization means that the GSE fails to meet both standards, but its core capital exceeds, respectively, 125.0 b.p. and 25.0 b.p. of on- and off-balance sheet assets. Should it fall below even that standard, it will be classified as "critically undercapitalized," and a conservator will be appointed to manage its operations, unless the director deems that such action would seriously destabilize financial markets.

Were it not for the disastrous experience with forbearance in the S&L debacle, this exception would not be so troubling. Unfortunately, the evidence is all too clear that regulators (and Congress) have every incentive to delay needed action in the hopes that any problem will resolve itself. This is especially true in the case of Freddie Mac and Fannie Mae, two institutions that, because of their status as GSEs, so dominate the market that they are widely believed to be too big to fail.

Whether or not the conditions established for the stress test are appropriate is debatable. The conditions suggested are severe. The question is simply whether those conditions cover enough of the distribution of possible economic states to adequately reduce the risk imposed on the taxpayer and the resulting subsidy conferred upon the GSE. In order to answer that question, a mechanism must be developed to estimate the value of the

[10] *Federal Housing Enterprises Financial Safety and Soundness Act of 1992*, P.L. 102-550, Oct. 28, 1992, pp. 3972–3973.

subsidy and estimate the sensitivity of that value to changes in the level of capital.

Valuation Methodology

In this section, the market value of the guarantee is estimated using a combination of an actuarial technique and the Black/Scholes option pricing model. The actuarial approach generates an estimate of the risk neutral value of insuring all obligations. This is used as a proxy for the value of a put option on all of the underlying real estate backing FHLMC liabilities, and the Black/Scholes model is used to extract the implied variance of those assets. The exercise price is then adjusted for the organization's ability to absorb losses from its own resources, and this is used with the previously calculated implied variance model to estimate the value of the federal guarantee.

The Option Pricing Approach

The federal guarantee of Freddie Mac obligations may be viewed as a second put option on mortgages financed directly or indirectly by Freddie Mac. The first put is created when Freddie Mac guarantees the payment of interest and principle on the mortgages that it securitizes. Alternatively, the first put may be considered as the aggregate of the put or default options held by mortgagees. Since a mortgage will not default or result in a loss to Freddie Mac unless the realizable market value of the real estate falls below the unpaid balance, this promise amounts to a warranty to the mortgagees that the price of the real estate will remain above the unpaid principle. The implicit put that the market believes has been issued by the federal government will be exercised only in the event that Freddie Mac is unable to honor the put that it has written. In the event of the default by Freddie Mac, it is expected that the federal government will purchase Freddie Mac securities from their holders at a price equal to the remaining face value plus accrued interest. This approach is equivalent to the one suggested by Hendershott and Van Order (1987). They argue that with 100.0% insurance coverage on the par (not market) value of the mortgage, the insurer pays the difference between the par value and net proceeds from the sale of the collateral. Thus, from the perspective of the insurer, the face value of the mortgage plus accrued interest is the extent of their risk exposure.

In principle, the value of the guarantee may be estimated using the Black/Scholes put option pricing expression,

$$P = Xe^{-rT} * N(-d_2) - S * N(-d_1) \quad (1)$$

where

$d_1 = \{ln(S/X) + (r + s^2/2)T\}/\sigma(T)^{.5}$,
$d_2 = d_1 - \sigma(T)^{.5}$,
P = the market value of the put option,
X = the exercise price of the option (market value of the loan portfolio),
S = the market value of the underlying real estate assets (UPB/LTV),
r = the risk free rate of interest,
T = the time remaining to expiration of the option,
$N(d_1)$ and $N(d_2)$ = the cumulative normal density function for the standard normal deviates d_1 and d_2, and
σ^2 = the variance of the rate of return of the underlying asset (volatility).

The option pricing approach has its limitations. Estimation requires that one establish values for X, S, r, T and σ^2. Of these variables S and r are, in principal, directly observable. Selecting values for T, σ^2 and X is more difficult. Given the implicit nature of the option, there is no mechanism to trigger the exercise and force a payout. Consequently, the terms of the guarantee, and thus the term of the option, are not easily defined. Attempts to solve this problem when applying the OPM to deposit insurance have assumed that the insurance policy is renewed subsequent to a periodic audit of the organization. The audit period is then used as a proxy for the term of the option.[11] While Freddie Mac has not been subject to a formal examination as have commercial banks and thrift institutions, Title XIII of FHEFSSA requires that the director of the Office of Federal Housing Enterprise Oversight within HUD conduct an annual assessment of the performance and condition of both FNMA and FHLMC. This requirement is used as the basis for justifying a one year term for the put option. Estimating σ^2, the volatility, is also problematic. In addition to the usual problem of nonstationarity of the probability distribution of asset values, reliable data on returns to insured assets over any time period are not available. The market value of the mortgage portfolio is estimated using an option-based model and the distribution of mortgage coupon rates found in

[11] See, for example, Marcus and Shaked 1984, Merton 1987, Ronn and Verma 1986 and Cook and Spellman 1992.

the Freddie Mac Annual Report.[12] Finally, the price generated by the OPM is based on the assumption that the option writer will perform as promised. It makes no allowance for the inability of the insurer to honor its claims. While there is no question about the *ability* of the federal government to honor the guarantee, there is growing uncertainty about its *willingness* to do so.

The Actuarial Approach

The actuarial model utilizes historical frequency and severity of defaults to develop an estimate of the expected losses and, from that, an estimate of the value of the guarantee. The implicit federal guarantee of GSE obligations is the same as property, casualty or life insurance where the insurer accepts a given underwriting risk in exchange for compensation. However, unlike privately underwritten insurance, the implicit nature of the guarantee means that the insured can only guess as to the circumstances and manner in which its claims will be honored in the event of default by the GSE. There is no specification of the conditions under which payment will be made, the time frame in which payment can be expected, or the extent to which the claim will be honored.

A private insurance contract uses the capacities of the insurer to supplement those of the insured under pre-specified circumstances. For a fee, the insured purchases coverage with the expectation that the reduction in risk of loss is worth the cost of the contract. In turn, the insurer must set the premium at a level that covers expected losses on a portfolio and provides an acceptable return on capital. The degree to which risk is reduced for the beneficiary is then a function of two factors—the terms of the contract and the financial stability of the insurer. For instance, a thinly capitalized insurer or one that is inadequately diversified will itself be prone to failure. *Ceteris paribus*, the more financially stable the insurer, the greater the premium that the insured should be willing to pay and that the insurer will need to charge to generate an adequate return to equity.

On balance, the rate charged by a private insurer should be a function of: (1) the expected losses facing the insured; (2) the ability of the insured to absorb a portion of the losses without recourse to the insurer; and (3) the

[12] The Freddie Mac 1993 Annual Report, p. 24, reports the coupon range for mortgages. Using these coupon rates, an option-based model is used to estimate the total market value of Freddie Mac mortgages retained and securitized. For a descritpion of the option-based model see Spahr and Sunderman (1992).

financial stability of the insurer. Unfortunately, the actuarial model provides a mechanism for valuing only the first of these factors. It provides no guidance for adjusting the premium to reflect the capitalization of the insured or the credibility of the insurer.

The Methodology

The shortcomings of the option pricing and actuarial techniques can be overcome with an amalgam of the two that capitalizes upon the strengths of each. The actuarial model is used to estimate a pure default premium based on Freddie Mac's historical record. That premium is equal to the base rate that would be charged by a risk neutral private insurer independent of the capitalization of the insurer, and the capitalization of Freddie Mac. This assumes that there is no possibility that the insurer will fail to honor its guarantee in the event that Freddie Mac defaults and that Freddie Mac can absorb none of the losses from its own resources.

In the second step, the actuarial premium becomes a proxy for the cumulative value of implicit put options on real estate financed by mortgages held or securitized by Freddie Mac. The value of these options is used in conjunction with the Black/Scholes model to estimate an implied variance for the underlying real estate. This implied variance is not equivalent to that on the individual mortgages used to back Freddie Mac securities. Rather, it is the effective variance of the realizable market value of the real estate collateral. The implied variance from the entire mortgage portfolio is not estimated because default losses have occurred only in those cases where LTVs' ratio are high. According to Freddie Mac, the bulk of losses are attributable to mortgages with LTVs in excess of 80.0%, and these represent approximately 20.0% of Freddie Mac mortgages during the period of study. This group is assumed to generate all observed default losses.

The third step uses this implied variance to estimate the value of the put option when the Freddie Mac's capitalization and available cash flows are taken into consideration. This is done by subtracting from the exercise price the sum of the book value of Freddie Mac equity, loan loss reserves and the annual cash flows available to offset default losses.

The resulting value of the put is the estimate of the annual value of the federal government guarantee of Freddie Mac obligations in light of its loss experience and resources available to absorb those losses. Implicit in this method is the assumption that the terms of the guarantee are rewritten every year subsequent to an audit of Freddie Mac's financial condition.

Estimating the Guarantee

The Pure Default Premium

An actuarial model for estimating the mean pure default premium (MPDP) for Freddie Mac must be consistent with the relatively unique nature of its risk. During most economic periods, the risk exposure of Freddie Mac and, therefore, the federal government, is similar to that of a company that insures against risks where losses are typically low in frequency, low in severity, but potentially catastrophic. This means that the probability that any one mortgage will default is very low and that the expected loss, given that a loan has defaulted, is small, but where there is the potential for a major loss.

The potential for catastrophic loss exists because of the presence of a positive correlation between the frequency and severity of loss. For instance, during periods of severe recession in the real estate market, the probability of default rises at the same time that declining market values cause the severity of loss, when default occurs, to be much greater. This positive covariance will result in episodes of high frequency, high severity and losses for the guarantor, in this case, Freddie Mac. The correlation may be due to spatial (time series) dependencies among defaults or to a direct relationship between frequency and severity. It is during such periods that Freddie Mac will be least likely to be able to meet its obligations, forcing the federal government to honor its implied guarantee and incur losses. Thus, expected government losses depend not only on the frequency and severity of losses, but also on the degree to which there is a positive correlation between them.

Unlike life and automobile insurance, it is unreasonable to assume that frequency and severity of Freddie Mac losses are independent. Default is a function of changes in the ability of a borrower to maintain monthly payments and the relationship between the market value of the mortgage and the unpaid balance on the loan. Under "normal" economic conditions, default will tend to be restricted to isolated instances of economic adversity that affect individual borrowers and idiosyncratic or regionally isolated declines in market values. In the absence of widely generalized economic distress, the equity position of the borrower will generally allow the lender to avoid losses in the event of foreclosure. Thus, both frequency and severity will be low. However, during a general economic decline, the interruption of borrower income will be more widespread as will be the reduction in property values, and both frequency and severity of default will rise as economic conditions deteriorate. Conversely, during economic booms, interruption of income will be less frequent relative to "normal" levels of

economic growth, and property values will tend to rise steadily. Both factors will reduce the frequency and severity of default. As a consequence, the correlation between frequency and severity most likely will be positive, possibly strongly so. Furthermore, since most economic cycles may last for more than one year, default risk may display a time or spatial dependency. High loss rates in one year may be a good indication of high loss rates in the next. Models for estimating insurance premia that allow for interdependence between frequency and severity have been developed by Spahr and Escolas (1986) and Spahr, Sunderman and Amalu (1991). These two models, with some modifications, are the basis for this analysis.

Economic states which would lead to government losses have not occurred on a nationwide basis during Freddie Mac's existence and are not likely to be frequent. Thus, the ability to survive such an episode cannot be determined directly and must be inferred from incomplete data. This is done by explicitly incorporating in the model the dependence between the frequency and severity of loss and by observing the losses to Freddie Mac in the Southwest during that region's protracted economic slump.

The proposed model considers the relevant parameters for each risk exposure unit (*i.e.*, a dollar of Freddie Mac asset at par) and the resulting aggregate pure risk premium for Freddie Mac. To establish the relationships, let:

z = frequency of mortgage defaults;
y = severity of mortgage defaults—loss per dollar of guaranteed mortgage;
x = pure risk premium per dollar of guaranteed mortgage;
T = total losses incurred in a population of insured risks per period;
$E(z)$ = the expected probability of a default (frequency) for an individual mortgage risk;
σ_z^2 = the variance of the frequency of default occurrence;
$E(y)$ = the expected amount of loss (severity) if a loss occurs, per dollar of guaranteed mortgage or MBS;
σ_y^2 = the variance of the loss severity per dollar of guaranteed mortgages;
$Cov(z,y)$ = the covariance between the frequency and severity of the individual risk units;
$E(x_i)$ = the expected mean pure premium (hereafter MPP) for the i^{th} individual risk unit per dollar of guaranteed mortgage;
σ_i^2 = the variance of MPP for the ith individual risk per dollar of guaranteed mortgage;
$Cov(i,j)$ = the covariance between the MPP of the i^{th} and j^{th} individual risks;
$E(T)$ = the expected total incurred loss for the population of guaranteed mortgages;
σ_T^2 = the variance of total incurred losses for the population; and

$\rho_{i,j}$ = the correlation between the losses on the i^{th} and j^{th} risks.

The mean and variance for individual pure risk premiums are:

$$E(x) = E(zy) = E(z)E(y) + Cov(z,y), \tag{2}$$

and

$$\sigma_x^2 = E(z)^2\sigma_y^2 + E(y)^2\sigma_z^2 + 2E(z)E(y)Cov(z,y).^{13} \tag{3}$$

For defaults on Freddie Mac mortgage assets, it is likely that the term $Cov(z,y)$ is positive (*e.g.*, the severity of the loss tends to be positively correlated with the frequency of defaults).

Ex post severity may be measured for individual defaults, however, *ex post* default frequency can be measured only for groups of loans. Consequently, the measure of severity must be an average for all defaults over the same subgroup for which frequency is calculated. The manner in which data are grouped for purposes of measuring frequency should be such that the information content of the resulting covariance is maximized. Since default frequency and severity should be a function of economic conditions, the covariance between frequency and severity will also be measured across time.

Equations (2) and (3) demonstrate the bias that may result from simply multiplying the average annual frequency and severity to obtain annual loss estimates. Because of the covariance between frequency and severity, either Equations (2) or (3) may be used to estimate annual losses or they may be

---

[13] Goodman (1960) reports the variance of the product of two dependent random variables is

$$\sigma_x^2 = E(z)^2\sigma_y^2 + E(y)^2\sigma_z^2 + 2E(z)E(y)E_{11} + 2E(z)E_{12} + 2E(y)E_{21} + E_{22} - E_{11}^2,$$

where

$E_{11} = Cov(z,y)$
$E_{12} = E(z - E(z))(y - E(y))^2$
$E_{21} = E(z - E(z))^2(y - E(y))$ and
$E_{22} = E(z - E(z))^2(y - E(y))^2$.

Given these relationships, it is clear that Equation (4). $s_x^2 = E(z)^2s_y^2 + E(y)^2s_z^2 + 2E(z)E(y)Cov(z,y)$, is a good approximation for determining the variance of the pure premium for a dollar of insurance coverage for an individual risk.

calculated individually for each year by multiplying the annual frequency times annual severity.

The total expected losses for the population of Freddie Mac obligations, each with a face value of $1, is a function of the par value of assets insured, N, and the expected pure risk premium.

$$E(T) = NE(x_i), \tag{4}$$

and

$$\sigma_T^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} Cov(i,j) \tag{5}$$

Boundaries for the value of s can be established by assuming that

$\sigma_i = \sigma_j$ for all i and j.

Then if $Cov(i,j)$ equals zero for $i \neq j$, Equation (5) becomes

$$\sigma_T^2 = \sum_{i=1}^{N} \sigma_i^2 = N\sigma_i^2 \tag{6}$$

If the correlation coefficient, $Cov(i,j)/\sigma_i\sigma_j$, equals one for all i and j, Equation (6) becomes

$$\sigma_T^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} \sigma_i^2\sigma_j^2 = N^2\sigma_i^2 \text{ [14]} \tag{7}$$

Assuming a first order autoregressive stationery process with $0 \leq \rho \leq 1$,

$$\sigma_T^2 = \sigma_i^2 J \begin{vmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{N-1} \\ \rho & 1 & \rho & \cdots & \rho^{N-2} \\ \rho^2 & \rho & 1 & \cdots & \rho^{N-3} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \rho^{N-1} & \rho^{N-2} & \rho^{N-3} & \cdots & 1 \end{vmatrix} J^T \tag{8}$$

where J is the N by 1 vector of ones.

[14] First order autocorrelation will be used as a proxy for the dependence of one risk unit being correlated to a second risk unit. Thus, $Cov(x_t,x_{t-1})$ is used as the proxy for $Cov(i,j)$ and $\rho_{ij} = Cov(x_t,x_{t-1})/\sigma_i^2$.

An estimate of Equation (8) is

$$\sigma_T^2 = N\sigma_i^2 + N\overline{M}\sigma_i^2 \tag{9}$$

where $\overline{M}$ is the average sum of each column of the correlation matrix in Equation (8) after taking out the diagonal of ones.[15]

If N represents the number of years observed in the sample, the population *MPP*, will be identical to the individual *MPP*.

$$E(\overline{X}) = \frac{E(T)}{N} = E(x) \tag{10}$$

Because of the Central Limit Theorem, $\overline{X}$ may be assumed to be normally distributed. This assumption is valid regardless of the underlying distribution of x or the covariance relationships if the distribution has a finite variance that is known or that can be estimated, and the sample size is sufficiently large to offset skewness in the underlying distribution. In this situation, sample size will not be a problem.

In an insurance context, the degree of confidence in measuring the mean pure default premium depends on the quality of historical loss data and the breadth of economic states that have been sampled. For samples taken over a short time period or where only a limited number of economic conditions have been sampled, a large confidence interval should be used. If there is a large sample of observations covering a wide range of possible states of nature, the confidence interval may be considerably smaller. Based on the approach taken here, the general expression for the percent confidence interval is given by Equation (11).

$$\overline{X} \pm z\sqrt{\sigma_i^2 \frac{(1 + \overline{M})}{N}} \tag{11}$$

where $\overline{X}$ is the estimate of μ_x and z varies with the size of the desired confidence interval.

[15] For Equations (4)–(7), N represents the total dollar value of guaranteed assets. However, since the first order autocorrelation for annual losses serves as a proxy for loss dependencies across individual risk units, N is the number of years observed in the sample for Equations (9)–(12).

The mean pure default premium for Freddie Mac is estimated using historical data on default and loss rates on Freddie Mac assets. The annual frequency of default, Equation (12), is equal to the par value (unpaid balance) of Freddie Mac assets that defaulted in each year divided by the par value (unpaid balance) of all Freddie Mac assets outstanding during the year.

$$Default\ Frequency = RVDB/PVAB, \tag{12}$$

where *PVDB* is the par value of defaulted Freddie Mac assets, and *PVAB* is the par value of all Freddie Mac assets.

The severity of loss for assets defaulting in any given year, Equation (13), is equal to actual losses on defaulted assets divided by the par value of those assets.

$$Loss\ Severity = ALBI/PVDB, \tag{13}$$

where *ALBI* is the actual losses to Freddie Mac for a default, and *PVDB* is the par value of defaulting assets.

Actuarial Data

Ideally, the analysis should be conducted for individual mortgages, and the sum of the values of the individual put options would represent the potential loss to Freddie Mac and the federal government. Unfortunately, the data necessary for that level of analysis were not available from Freddie Mac. Aggregate data on default experience were provided by Freddie Mac's headquarters in Virginia. These data cover loss rates for the entire U.S. and for each of the five Freddie Mac regions. The original data were classified according to the year (1976–1989) in which the mortgages originated and by the year (1983–1989) in which the properties went into default. The years selected were based purely on the availability of data. Freddie Mac's data base did not contain the necessary information on mortgages that originated prior to 1976 or defaulted prior to 1983. Ultimately, data on mortgages that originated prior to 1979 and/or that defaulted later than 1988 were not utilized to estimate the default premium. Default data was not available prior to 1983, consequently, mortgages originating prior to 1979 were judged to be too old to be included in the sample. The bulk of the defaults tend to occur early in the life of the mortgage and defaults on older mortgages tend to have lower losses. Conversely, mortgages originating after 1984 were excluded because they were judged not to have completed enough of their life cycle to be representative. According to HUD's *Report to Congress on the Federal Home Loan Mortgage Corporation*, December 1992, mortgages

that originated in 1980 through 1983 had the highest default rates in Freddie Mac's history. Consequently, the default losses observed in this study may be higher than normal. The sample covered 29,323 loans. Officials at Freddie Mac chose not to include data on defaulted mortgages where the amounts realized on the sale of the collateral were less than 50.0% or more than 150.0% of the unpaid balance. No information on the number or dollar value of such loans was provided, and no explanation for their exclusion was given except that they were extremely rare and aberrant cases.

The loss associated with each default is defined as the unpaid balance of the loan plus the present value (at the date of failure) of all expenses incurred in liquidating the collateral less the present value of all cash inflows. Liquidation expenses include lost interest and legal and administrative expenses associated with the foreclosure. Cash inflows include proceeds from the sale of the collateral, private mortgage insurance, FHA/VA insurance, rental income and miscellaneous items. Expenses, rental income, and other proceeds were assumed to occur halfway between the REO or acquisition date and the settlement date. All other flows were discounted from the date at which they occurred. Sale proceeds were discounted at 15.0%; all other flows at 10.0%.

Obviously, both the frequency and severity of defaults are dependent on the economy of the region in which the defaults occurred. For example, the recession in the Southwest caused a large increase in both frequency and severity of defaults. By segmenting the data according to region (Southwest, Northeast, etc.), defaults were measured for a wider set of economic states than would have been possible if the data had been aggregated nationally.

Actuarial Estimates

As of the end of 1993, Freddie Mac directly or indirectly held mortgages with principals of $494,505 million, of which $439,029 million were financed with MBSs and $55,476 million by various types of debt including multi-class securities.[16] These assets constitute the basis on which insurance premiums are calculated. Estimates of default frequency and severity, and the correlation between them, are based on the historical experience of Freddie Mac for mortgages originating from 1979 through 1984 which defaulted from 1983 through 1988. The data on default and severity have been organized by region and year of origination. A sample of those data,

[16] *Information Statement*, March 31, 1994, p. A-19. Figures are rounded to the nearest $100 million.

Table 1 ■ FHLMC Default experience.

Region	REO Year	Individual Average Frequency Z (%)	Group Average Severity Y (%)	Severity $Y1$ (%)	Loss b.p. X	Loss b.p. $X1$
AL	1983	1.3	21.3	21.1	26.8	27.6
AL	1984	0.7	26.4	24.9	19.5	18.4
AL	1985	0.7	28.1	25.5	19.0	17.3
AL	1986	1.1	33.7	31.3	37.4	34.7
AL	1987	1.9	42.4	38.3	81.9	73.0
AL	1988	1.9	39.5	34.9	76.7	67.8
Mean		1.3	31.9	29.3	43.4	39.6
Std. Dev.		0.5	7.4	6.0	25.8	22.6

All regions have a 1982 origination. Corr (Z, Y) is 0.75, Serial Corr (X) is 0.65 and 95% CI = (0, 119.2 b.p.). Corr (Z, $Y1$) is 0.76, Serial Corr ($X1$) is 0.64 and 95% CI = (0, 105.9 b.p.). The second column is the year of default, the third is the proportion of mortgage book value that defaulted, the fourth is the average severity of loss, the fifth is severity, defined as the present value of total losses divided by the unpaid balance for that loan class at time of default, and the sixth and seventh are the products of the third, fourth and fifth, respectively.

the resulting *MPP*, and related information are presented in Tables 1 and 2 for the entire U.S. and the Southwest, respectively. These data cover mortgages that originated in 1982.

Interpretation of the tables is straight forward. The third column indicates the year the property went into default. The fourth column contains the frequency of default. As noted earlier, it is calculated by dividing the par value of the unpaid balance of all loans that went into default (both REO and nonREO) by the dollar value of the unpaid balance of all loans of that class. Column five is an equally weighted average severity of loss for individual loans as calculated by Freddie Mac. Column six contains a value weighted measure of severity defined as the present value of total losses divided by the total unpaid balance for that loan class. Because there is very high correlation between the two weighting schemes, the equally weighted measure of severity is used. Columns seven and eight are the products of columns four and five and four and six, respectively.

Because default rates are not available by individual mortgages, the rates are based on groups of mortgages. Table 1 presents data on a group of mortgages that originated in 1982 and defaulted in one of the years 1983–1988. The correlation coefficients in Tables 1 and 2 reflect the relationship between

Table 2 ■ FHLMC Default experience.

Region	REO Year	Individual Average Frequency Z (%)	Group Average Severity Y (%)	Severity $Y1$ (%)	Loss b.p. X	Loss b.p. $X1$
SW	1983	0.7	23.2	21.0	17.0	15.4
SW	1984	1.2	38.1	26.8	44.8	31.5
SW	1985	1.8	30.1	27.5	52.8	48.2
SW	1986	3.4	43.7	41.4	149.2	141.5
SW	1987	6.1	49.7	45.1	302.7	274.9
SW	1988	5.2	40.7	37.7	211.4	195.7
Mean		3.1	37.6	33.3	129.7	117.9
Std. Dev.		2.0	8.7	8.7	102.4	95.9

All regions have a 1982 origination. Corr (Z, Y) is 0.83, Serial Corr (X) is 0.61 and 95% CI = (0, 430.2 b.p.). Corr (Z, $Y1$) is 0.92, Serial Corr ($X1$) 0.65 and 95% CI = (0, 396.7 b.p.). The second column is the year of default, the third is the proportion that defaulted, the fourth is the average severity of loss, the fifth is severity, defined as the present value of total losses divided by the unpaid balance for that loan class at time of default, and the sixth and seventh are the products of the third, fourth and fifth, respectively.

frequency and severity among 1982 originations according to year of default. The mean and standard deviations of X and $X1$ are the equivalent of Equations (2) and (3) for alternative measures of severity, Y and $Y1$. The average loss severity Y, 31.9%, yields a *MPP* for Freddie Mac mortgages originating in 1982 of 43.37 b.p. with a standard deviation of 25.8 b.p. Given a serial correlation of 0.65. the 95% confidence interval produced by Equation (12) is bounded by 0 and 0.01.

Equivalent data are presented for the Southwest region in Table 2 where an average severity of 37.6% yields a *MPP* of 129.7 b.p., a standard deviation of 102.3 b.p. and, based again on Equation (12), a 95% confidence interval of 0.0 to 430.2 b.p. For the Southwest, the *MPP* is slightly above the upper bound on the 95% confidence level for the entire country, reflecting the severity of the depression in the real estate market in that region during the period studied.

Table 3 summarizes the results for Freddie Mac's nationwide default experience on mortgages originating in 1979–1984 and defaulting in 1983–1988. Based on these data, the *MPP* for Freddie Mac is 27.0 b.p. Given the limited number of states of nature covered by the sample period, it can be argued that the nationwide MPP is not an acceptable basis for

Table 3 ■ National default experience.

Year	$E(X)$ b.p.	Std. Dev. $E(X)$	Average Mortgage Market Value ($000)[17]
1979	9.7	2.8	96.11
1980	19.5	5.7	102.85
1981	37.8	12.4	108.08
1982	43.4	18.7	109.52
1983	26.7	15.8	102.53
1984	25.5	15.7	105.40
Average	27.1	11.9	104.08
Std. Dev.	12.2		

The first column is year of origination, the second is the mean pure premium for that group, the third is the standard deviation of the mean pure premium and the fourth is the average market value of mortgages.

estimating the value of Freddie Mac's guarantee. A more conservative approach would be to use the 95% confidence interval or 65.0 b.p. Given outstanding obligations of $494,510 million, these results translate to an annual premium of $1,339 million using the 27.0 b.p. *MPP* or $3,232 million using the 95% confidence interval of 65.0 b.p.

By way of comparison, Table 4 presents comparable data on the Southwest region. These results might realistically be considered as the worst case scenario should the experience there be repeated nationwide. The differences are dramatic, and while not indicative of expected losses to Freddie Mac, they are a powerful demonstration of the value of geographic diversification. The mean premium of 83.0 b.p. is triple that for the nation as a whole and the 249.0 b.p. premium for the 95% confidence interval is almost four times as large as the comparable nationwide estimate. If the losses implied by the average experience in the Southwest were to be repeated nationwide, Freddie Mac annual default losses would be $4,122 million. Again, the time period covered in this study had the highest default rates in Freddie Mac's history. This most likely is a result of the severe problems in the Southwest and higher than average defaults in the West. From the analysis and from HUD's *Report to Congress on the Federal Home Loan Mortgage Corporation*, December 1992, 10.0% of mortgages were from the Southwest region and

[17] Using an option-based model and data on mortgage coupon rates, maturities, option-adjusted spreads and the U.S. Treasury term structure, estimates of the market price for defaulting mortgages for each year of origination were made. The average price ($104.084) is the weighted average price for defaulting mortgages over the 1983–88 time period.

Table 4 ■ Southwest default experience.

Original Year	$E(X)$ b.p.	Std. Dev. (X)
1979	25.5	15.7
1980	46.0	31.6
1981	78.2	53.5
1982	129.7	102.3
1983	107.4	63.4
1984	108.5	72.6
Average	82.6	55.7
Std. Dev.	49.0	

The first column is the year of origination, the second is the mean pure premium for that group and the third is the standard deviation of the mean pure premium.

63.0% were from the West. The estimated default loss of 15.0 b.p. is still higher than the 1992 HUD report and the losses estimated by Quigley and Van Order (1991). They found that the average default rate was 0.4% and average losses were 13.7 b.p. for loans that originated between 1976 and 1980 and observed through 1989. Without the Southwest loss experience and the high concentration of West region loans, the average default losses would be less than 15.0 b.p.

Subsequent calculations will use a range of values for the *MPP* as the basis for estimating the value of the guarantee. It is worth repeating that the *MPP* is based only on Freddie Mac's historical losses during this subperiod and does not reflect either Freddie Mac's ability to absorb losses or financial stability of the insurer. The historical losses are the basis for determining the *MPP* that would be charged by a risk neutral insurer that has no default risk of its own when insuring an entity that cannot to absorb losses from its own resources.

The Implied Variance

The second step in estimating the value of the guarantee is to use the actuarially determined *MPP* to develop an option's implied variance of return on Freddie Mac's assets. The *MPP* estimated in the previous section is multiplied by the book value of mortgages currently held or securitized in Freddie Mac's portfolio. This product is the imputed loss. The imputed

loss is the price that a risk neutral insurer would charge to guarantee the value of that real estate. It is an estimate of the value of the combined Freddie Mac and federal government guaranties. The strike price of the hypothetical put option is the market value of at-risk mortgages. The market value of the real estate financed by such mortgages is estimated by dividing their unpaid balance by their average LTV.[18]

It would be preferable to estimate the *MPP* and implied variance for separate LTV classes of mortgages since the probability of default varies positively with LTV ratios. Those with very low ratios simply do not default. Since the data are not sufficiently disaggregated, the approach taken here will be to assume that the bulk of default losses will be from the group of mortgages with market or spot LTV ratios in excess of 80.0%. Using the variance implied by observed default losses and the market value for mortgages held and securitized by Freddie Mac in December 1993, the put value of mortgages as a function of market LTV is as follows:

LTV	1.00	0.90	0.80	0.70	0.60
PUT VALUE	530 b.p.	192 b.p.	41 b.p.	4 b.p.	0 b.p.

Given these results, mortgages are classified to be at risk or most likely to default if their market LTV equals or exceeds 80.0%. This classification is confirmed by Freddie Mac's *Information Statement*, in which it is observed that mortgages with origination LTVs in excess of 80% have a default experience that is significantly less favorable than those below. These more risky mortgages with market LTVs greater than 80.0% constituted approximately 20.0% of its sold and retained portfolio.[19] Data from Freddie Mac also indicate that only 4.0% of its loans had current market LTVs in excess of 95%. Assuming a beta distribution for current market LTVs in the at-risk class, the mean current market LTV for that group is estimated to be 87.3%. At 20.0% of Freddie Mac's portfolio, the at-risk class has a 1993

[18] Freddie Mac's estimate of its average LTV as of December 31, 1993 was 0.66, *Information Statement*, March 31, 1994, p. 32. As of December 31, 1993 the book value of mortgages financed by Freddie Mac was $494,505 million, yielding an imputed market value of the underlying real estate of $749,250 million. Ideally, the LTV would be contemporaneous with the period during which the mean pure premium was estimated. Unfortunately such data are not available. The most recent figures were used in response to a suggestion by Freddie Mac that earlier data underestimated the true LTV thus imparting a positive bias to the implied variance. Also, in an option framework estimating the value of default, the market value of the mortgage is commonly used as the exercise price of the put option (see Hendershott and Van Order 1987, Kau and Keenan 1995, Vandell 1995 and Shilling 1995).

[19] *Information Statement*, March 31, 1994, p. 14.

book value of $98,901 million, an estimated market value of $103,392 million and, using the market LTV ratio of 87.3%, the estimated market value of the collateral is $113,250 million.[20]

Five different estimates of the *MPP* are used to estimate the implied variance—35.0, 27.0, 18.0, 15.0 and 8.0 b.p. The 35.0 b.p. *MPP* is one standard deviation above the mean of 27 b.p.. The 18 b.p. MPP is the weighted average of the 15.0 b.p. *MPP* that excludes the Southwest experience (weight = 0.95) and the 83.0 b.p. *MPP* for the Southwest alone (weight = 0.05). The 8.0 b.p. *MPP* is an arbitrarily low value to reflect exceptionally strong economic conditions. Imputed losses on the portfolio are calculated by multiplying each *MPP* by $494,510 million, the December 31, 1993 book value of Freddie Mac purchased mortgages. The imputed losses determine the premium charged by a risk neutral insurer to cover losses expected on the at-risk loans or the value of the put option on those loans. The market value of the mortgage portfolio is the strike price of the put. Combining this with the estimated market value of the underlying real estate, a twelve month expiration for the option, and a risk free rate of 6.0%, the Black/Scholes option model yields the implied variances (annualized) that are presented in Table 5.

Estimation of the Value of the Federal Guarantee

In order to separate the value of the federal guarantee from the combined Freddie Mac and federal guarantee, the degree of protection provided by the combination of Freddie Mac capital, loan loss reserves and cash flows is estimated. The exercise price for the federal government is effectively reduced by the value of these resources available to Freddie Mac. The put option written by the federal government will not be exercised until the resources available to Freddie Mac are exhausted. This new strike price, the previously estimated implied variance and risk free rate are used as inputs to the OPM to determine the value of the government guarantee. At the end of 1993, the market value of at-risk loans was $103,392 million and the loss that could be absorbed by Freddie Mac was $6,197 million, leading to an exercise price of the put of $97,195 million. Estimates of the federal

[20] *HUD's Report to Congress on the Federal Home Loan Mortgage Corporation*, December 1992, p. 33, reports that the initial load-to-value ratio for 14.0% of Freddie Mac mortgages was over 90.0% in 1981 as opposed to only 6.0% of mortgages having an initial LTV of over 90.0% in 1991. The corresponding percentages for the 80.0%–90.0% LTV category were 35.0% in 1981 and 16.0% in 1991. Because of LTV being a critical variable in the assessment of default risk, this reduction in LTV from 1981 to 1991 represents a substantial reduction in risk.

Table 5 ■ Estimates of the implied standard deviation.

Total *MPP* (b.p.)	Value of Mortgages at Risk ($ millions)	Implied Value of R.E. Backing Mortgages ($ millions)	Imputed Losses ($ millions)	Implied Std. Dev. (%)
35.0	103,391	113,250	1,731	4.8
27.1	103,391	113,250	1,339	4.3
18.0	103,391	113,250	890	3.8
15.0	103,391	113,250	740	3.6
8.0	103,391	113,250	400	2.0

LTV: 87.3%, 20% of mortgages at risk and average market value of mortgages of $104,541. The first column contains different estimates of the pure premium; the second is the estimate of the market value of at risk mortgages; the third is the implied market value of real estate backing the mortgages; the fourth is the imputed losses; and the fifth is the implied standard deviation.

guarantee are presented in Table 6 for the range of *MPP*s, implied variances, and loan to value ratios found in Table 5. All entries in Table 6 assume an average current LTV for at-risk loans of 87.3%.

These results suggest that the federal government bears no more than 50% of the total risk of insuring mortgages financed by Freddie Mac given the financial structure and economic conditions as of the end of 1993. The

Table 5A ■ Estimates of the implied standard deviation.

Total *MPP* (b.p.)	Value of Mortgages at Risk ($ millions)	Implied Value of RE Backing Mortgages ($ millions)	Imputed Losses ($ millions)	Implied Std. Dev. (%)
82.6	258,481	283,125	4,084	4.7
35.0	258,481	283,125	1,731	3.5
27.1	258,481	283,125	1,339	3.3
18.0	258,481	283,125	890	3.0
15.0	258,481	283,125	742	2.9
8.0	258,481	283,125	396	2.5

LTV: 87.3%, 50.0% of mortgages at risk and average market value of mortgages of $104,541. The first column contains different estimates of the pure premium, the second is the estimate of the market value of at risk mortgages, the third is the implied market value of real estate backing the mortgages, the fourth is the imputed losses and the fifth is the implied standard deviation.

Table 6 ■ Value of the implicit government guarantee.

Total *MPP* (b.p.)	Implied Std. Dev. (%)	Exercise Value ($ million)	Government Guarantee ($ million)	Government Guarantee (b.p.)	Government Guarantee/ Total *MPP* (%)
35.0	4.8	97,195	802	16.2	46.3
27.1	4.4	97,195	560	11.3	41.8
18.0	3.8	97,195	311	6.3	34.9
15.0	3.6	97,195	238	4.8	32.1
8.0	2.0	97,195	2	4.0	5.1

LTV: 87.3%, 20.0% of mortgages at risk and average market value of mortgages of $104,541. The first two columns are the MPP and associated implied standard deviation, the third is the exercise value of the put option, the fourth is the dollar value of the government guarantee, the fifth is the government guarantee as a proportion of the book value of purchased mortgages and the sixth is the government guarantee as a percentage of the mean pure premium.

positive relationship between the federal guarantee and both the MPP and implied variance is consistent with expectations. These estimates are 102.0%, 71.2%, 39.6%, 30.3% and 0.3%, respectively of Freddie Mac's 1993 net income of $786 million. These estimates are representative of the state of the mortgage market and Freddie Mac's financial structure at the end of 1993. Should any of these variables change, the value of the guarantee would change as well. At the extreme, the federal government may be liable for

Table 6A ■ Value of the implicit government guarantee.

Total *MPP* (b.p.)	Implied Std. Dev. (%)	Exercise Value ($ million)	Government Guarantee ($ million)	Government Guarantee (b.p.)	Government Guarantee/ Total *MPP* (%)
82.6	4.7	252,284	3,031	61.3	74.2
35.0	3.5	252,284	1,119	22.6	64.7
27.1	3.3	252,284	827	16.7	61.8
18.0	3.0	252,284	509	10.3	57.2
15.0	2.9	252,284	409	8.3	55.1
8.0	2.5	252,284	191	3.9	48.3

The first two columns are the MPP and associated implied standard devaition; the third is the exercise value of the put option; the fourth is the dollar value of the government guarantee; the fifth is the government guarantee as a proportion of the book value of purchased mortgages; and the sixth is the government guarantee as a percentage of the mean pur premium.

the entire book value of mortgages financed, since Freddie Mac does not have the resources to withstand a truly catastrophic decline in the market values of real estate.

To get some idea of the impact of different economic conditions, Tables 5A and 6A repeat the previous calculations for circumstances in which 50.0% of Freddie Mac mortgages have current LTVs in excess of 80.0% and the value of the highest pure premium (82.6 b.p.) is equal to that experienced in the Southwest for the period covered by this study. Table 6A presents the value of the federal government implied guarantee under the scenario for Table 5A. The value of the guarantee is substantially larger, rising to a maximum of $3,031 million annually or 61.3 b.p. The risk borne by the federal government represents over 74.0% of the total risk of Freddie Mac. Even for the 15.0 b.p. estimate of the MPP, the federal guarantee is $490 million annually or 55.0% of the total risk. The value of the estimates relative to 1993 net income are 385.2%, 142.4%, 105.2%, 64.8%, 52.0% and 24.3%, respectively. It is clear that the value of the federal government implicit guarantee is very sensitive to the state of the economy and the real estate market as represented by the distribution of LTVs.

As previously noted, an overall measure of the default risk portion of the guarantee can be constructed by using a weighted average of the guarantees for the MPP most appropriate for the specific economic conditions. Assigning a probability of 0.95 for "normal" conditions and 0.05 for extreme adversity, and using *MPP*s of 15.0 and 83.0 b.p., respectively, the value of the federal guarantee is estimated to be 7.6 b.p. or $377 million for 1993. This is equal to 48.0% of Freddie Mac's 1993 net income.

The Cost of Default and the Exercise Price

In theory, the perfectly rational borrower should default as soon as the market value of the collateral falls below the market value of the mortgage. In practice, default occurs only when the market value of the real estate falls substantially below the market value of the mortgage. Foster and Van Order (1984) find that even when equity is negative (LTV $>$ 1.0), the probability of default is still less than 10.0%. In a sample of 1,191 defaults at a failed Northeastern thrift, Crawford and Rosenblatt (1995) find that the average ratio of market value to unpaid balance was 78.1% (LTV = 1.3) at the time the property was sold. The ratio at the time of default was not reported. This delay is due to transactions costs borne by the borrower in the event of default (Foster and Van Order 1984, Hendershott and Van Order 1987, Crawford and Rosenblatt 1995 and Ambrose, Buttimer and Capone 1995). These costs include, but are not limited to, those associated with moving,

damage to credit standing, potential tax liabilities and recourse to personal assets.

The presence of positive borrower costs of default is equivalent to an increase in the market price of the underlying real estate by an amount equal to those costs. Consequently, the value of the borrower's perceived put option is *lower* than the intrinsic cost of the option for the insurer. For an insurer such as Freddie Mac, this asymmetry is favorable for it reduces the default losses for a given variance of property values. It also means that for any given default experience, the variance in property values must be greater than would be the case in the absence of default costs. Since the implied variance is calculated assuming default as soon as equity turns negative, the estimates contain a negative bias. This in turn imparts a negative bias to the estimate of the value of the federal guarantee.

While the sign of this bias is clear, its value is not known. All that can be done is to examine the sensitivity of both the implied variance and guarantee to the presence of borrower default costs. For instance, if these costs are 5.0% of the market value of the real estate, the implied standard deviation for the 27.0 b.p. Pure premium increases from 4.4%–5.1% and the federal guarantee from $560 to $632 million. Costs of 10.0% result in an implied standard deviation of 6.0% and a guarantee of $686 million. Clearly, while the guarantee is sensitive to these costs, it is not so sensitive as to change the order of magnitude of the base estimates.

The Likely Effectiveness of FHEFSSA Capital Requirements

The basic premise of FHEFSSA is that Freddie Mac and Fannie Mae should maintain an adequate capital base to protect the federal government against the risk of default. Given the December 1993 distribution of on- and off-balance sheet holdings of mortgages, Freddie Mac is more than adequately capitalized according to FHEFSSA's standards. The minimum primary capital required by FHEFSSA is $4,380 million, while the actual primary capital is $5,197 million. Nevertheless, Tables 6 and 6A demonstrate that even with a capital base that exceeds the minimum requirement by almost 19.0%, Freddie Mac still enjoys a federal guarantee worth $377 million, an amount equal to 48.0% of its 1993 net income.

Clearly, FHEFSSA has not eliminated the taxpayer subsidy. The primary reason for this is the implicit assumption that interest rate risk is the most important source of risk for a housing GSE. This assumption is evident in the lower FHEFSSA capital standard for off-balance sheet mortgages. The results of the methodology here, which focuses exclusively on credit risk,

clearly show that credit risk is the primary factor determining the value of the federal guarantee.

In order to demonstrate the sensitivity of Freddie Mac's subsidy to additional primary capital, the value of the guarantee is estimated assuming that primary capital is increased by $9,803 million to $15,000 million or 3.0% of on- and off-balance sheet mortgages. The additional capital would reduce the exercise price of the federal government's short put. The results presented in Table 7 are for "normal" economic conditions; those in Table 7A are for a severe real estate slump comparable to the Southwest experience. The increase in capital under normal conditions will cause a substantial decline in the guarantee. For the 35.0 b.p. *MPP*, the subsidy falls from 16.2 b.p. to 4.1 b.p. For the 15.0 b.p. *MPP*, the subsidy declines from 4.8 b.p. to 0.6 b.p. On the other hand, Table 7A presents a much less optimistic picture. Here, the government guarantee is still 62.7% of that in Table 6A for a *MPP* of 82.6%, and 38.3% of that for an *MPP* of 15.0. Taking a weighted average of the basis point subsidy under the normal and disaster scenarios, primary capital to 3.0% of on- and off-balance sheet mortgages will reduce the federal guarantee to roughly 2.5 b.p. or $122 million, 15.5% of FHLMC net income for 1993.

This suggests that increased capital requirements can significantly reduce the default risk imposed on the taxpayer by the operations of Freddie Mac and, by implication, Fannie Mae. However, the capital requirements imposed by FHEFSSA are grossly insufficient.

Table 7 ■ Value of the implicit government guarantee at 3.0% FHLMC capitalization.

Total *MPP* (b.p.)	Implied Std. Dev. (%)	Exercise Value ($ million)	Government Guarantee ($ million)	Government Guarantee (b.p.)	Government Guarantee/ Total *MPP* (%)
35.0	4.8	88,392	201	4.1	11.6
27.1	4.4	88,392	115	2.3	8.6
18.0	3.8	88,392	44	0.9	4.9
15.0	3.6	88,392	28	0.6	3.1
8.0	2.0	88,392	0	0.0	0.0

LTV: 87.3%, 20.0% of mortgages at risk. The first two columns are the *MPP* and associated implied standard deviation, the third shows the lower exercise price due to the greater capitalizations; the fourth through sixth are recalculations based on the new value of the third.

Table 7A ■ Value of the implicit government guarantee at 3.0% FHLMC capitalization.

Total *MPP* (b.p.)	Implied Std. Dev. (%)	Exercise Value ($ million)	Government Guarantee ($ million)	Government Guarantee (b.p.)	Government Guarantee/ Total *MPP* (%)
82.6	4.7	243,481	1,902	38.5	46.6
35.0	3.5	243,481	560	11.3	32.3
27.1	3.3	243,481	384	7.8	28.7
18.0	3.0	243,481	207	4.2	23.3
15.0	2.9	243,481	157	3.2	21.1
8.0	2.5	243,481	59	1.2	14.9

(LTV: 87.3%, 50.0% of mortgages at risk. The first two columns are the *MPP* and associated implied standard deviation, the third shows the lower exercise price due to the greater capitalization and the fourth through sixth are recalculations based on the new value of the third.

The evidence on default rates in this study are somewhat different from those reported by Freddie Mac itself and by the GAO (1990). The GAO study estimates default losses as a percentage of principal to be approximately 6.0% for 1985–1989.[21] This underestimates potential losses on principal because it divides total default experience in each year by the total of on- and off-balance sheet mortgages for that same year. For instance, the 1989 principal balance and default experience of Freddie Mac were $294,722 million and $207 million, respectively. Using the GAO methodology, this yields a default rate of 7.0 b.p. According to Freddie Mac, mortgage defaults have a life cycle where losses are low initially, increasing until the loan is approximately five years old and then dropping off for the remainder of the mortgage's life. Thus, it is inappropriate to compare the losses in 1989 to the mortgage principal balance in 1989, as was done in the GAO study. Those mortgages that originated in 1988 and 1989 (a large portion of a growing portfolio) have not reached the stage of their life cycle where most defaults occur.

A more appropriate measure of default losses would be to estimate the weighted average origination year for defaulting mortgages and divide the losses by the principal balance of mortgages for that origination year. Assuming that the average year of origination for mortgages defaulting in

[21] United States General Accounting Office, 1990, *Government Sponsored Enterprises: The Government's Exposure to Risk*, Washington, D.C., p. 52

1989 is 1985, the loan losses in 1989 as a percentage of 1985 principal is 18.0 b.p. or three times the level indicated in the GAO report. The 18.0 b.p. estimate is comparable to the results reported here when the Southwest experience and the high proportion of Freddie Mac loans found in the Western region are compensated for.

Policy Alternatives

There are four alternatives that can be used to deal with the subsidy received by housing GSEs: (1) privatization; (2) user fees; (3) increased capital requirements; and (4) accept the subsidy and report it as an expenditure or a contingent liability in the federal budget. The choice depends upon societal goals. If the goal is simply to structure GSEs to eliminate the implicit subsidy, the most straight forward solution is to privatize them by eliminating their federal charter, all preferential treatment by public authorities and removing from their board all publicly appointed directors. The long-run consequences of this action cannot be assessed fully here, but as a first approximation such a step will certainly raise the cost of finance for the housing GSEs, exposing them to increased competition from private mortgage securitizers, and possibly raising the cost of mortgage credit nationwide. Given the earlier estimates, the potential increase in mortgage rates might be as high as 8.3 b.p.

If the goal is to maintain a stabilizing presence of the federal government in the housing market by allowing Freddie Mac and Fannie Mae to retain their GSE status, there are two alternatives for reducing the cost of the subsidy—higher capital requirements or insurance fees. Both will cause GSE costs to rise, leading to increased private sector competition and increased mortgage rates. The results presented in Tables 7 and 7A would suggest that higher primary capital requirements would be effective. The estimates indicate that an increase in primary capital to 3.0% of assets leads to a 4.2 b.p. reduction in the guarantee. In return for allocating $9.80 billion in additional capital to Freddie Mac, society would reduce the burden of the implicit guarantee by $.21 billion. The social NPV of this decision depends upon the discount rate used, however, the long-term real cost of debt to the Treasury and to the corporate sector are both below 2.5%. If so, this makes the present value of the reduced subsidy greater than the additional capital required.

Guarantee fees equal to the value of the subsidy would have the same effect as increasing capital requirements. In calendar year 1993, Freddie Mac reported net income of $786 million, net mortgages of $55,476 million and securitized mortgages of $439,029 million. This constitutes an annualized

return on mortgages of 15.9 b.p. Using 8.3 b.p. as a minimum value of the explicit guarantee, this requirement would force Freddie Mac to raise its 1993 management fee of approximately 23.7 b.p. to 32.0 b.p. If the guarantee fee levied is an unbiased estimate of the subsidy's market value, the impact of this option would be similar to privatization. However, charging a guarantee fee may be preferred when compare to privatizing since the federal government's explicit guarantee may be a stabilizing force in the home mortgage market—especially during periods of economic distress. Thus, if society's goal is to stabilize the home mortgage market, charging a guarantee fee in exchange for an explicit guarantee may be preferred to privatizing housing GSEs.

The fourth course of action is simply to recognize the cost of the subsidy on the federal budget. The subsidy could be justified as a means to stabilize the mortgage market and/or to lower mortgage financing costs. As major players in the mortgage securitization market, Freddie Mac and Fannie Mae have had primary responsibility for the greater liquidity and availability of mortgage capital. The implicit federal guarantee and the subsidy associated with it have played a major role in fostering the growth of these GSEs and the secondary mortgage market. While the private sector certainly could have created the market without the federal subsidy, it is probable that the growth would not have been as rapid. Thus, the benefits of the subsidy are distributed between two groups, private sector borrowers who enjoy lower mortgage rates and the owners and managers of the housing GSEs who operate in a market with competition limited by the federal subsidy. While the nature of the distribution is unknown, one course of action available to the federal government is to recognize that the subsidy exists, but justify it on the basis of its impact on the mortgage market.

Conclusion

This study has examined the nature of the implicit guarantee of government sponsored enterprises in general and Freddie Mac in particular and has developed a methodology for estimating the value of the guarantee against default loss for Freddie Mac. Since only 11.2% of Freddie Mac's mortgage financing is on-balance sheet, 88.8% of its mortgage financing is subject only to default risk. Interest rate risk may not be as significant as credit risk since it does not involve cash outflows. The greatest risk associated with insolvency due to adverse interest rate movements may be the agency cost inherent in a declining net worth of an insured intermediary.

A methodology was developed for estimating the guarantee against default losses based on a combination of actuarial and option pricing models. The

methodology yields an estimate of the value of the government's subsidy of Freddie Mac of 8.3 b.p., or roughly $410 million for 1993. Whether this should be eliminated by privatization, or increased capital requirements, charged directly to Freddie Mac in the form of a "user's fee," or simply recorded in the federal budget as part of the implicit cost of subsidizing the housing market, is subject to debate. Any of the first three courses of action will probably cause residential mortgage rates to rise, increasing private sector competition, but reducing the indirect subsidy to the home owner. Whether opportunities for efficiency gains from increased competition are sufficient to offset the lost subsidy is unknown. The choice among the alternatives is a function of the goals and objectives of society.

References

Ambrose, B.W., R.J. Buttimer, Jr. and C.A. Capone. 1995. The Impact of the Delay Between Default and Foreclosure on the Value of Mortgage Default. Working paper presented at the American Real Estate Society Conference: Hilton Head, SC.

Caso, E.S. Jr. 1990. *Fannie Mae Freddie Mac*. Goldman Sachs: New York, NY.

Cook, D.O. and L.J. Spellman. 1992. Taxpayer Resistance, Guarantee Uncertainty, and Housing Finance Subsidies. *Journal of Real Estate Finance and Economics* 5(2): 181–195.

Crawford, G.W. and E. Rosenblatt. 1995. Efficient Mortgage Default Option Exercise: Evidence from Loan Loss Severity. Working paper presented at the American Real Estate Society Conference: Hilton Head, SC.

Financial Institutions Reform Recovery and Enforcement Act, 1989, Public Law 101-101-73: 431.

Foster, C.C. and R. Van Order. 1984. An Options-Based Model of Mortgage Default. *Housing Finance Review* 4(4): 351–372.

Gatti, J.F. and R.W. Spahr. 1992. The Burden of Government Sponsored Enterprises: The Case of Freddie Mac. *Cato Public Policy Studies*. Cato Institute: Washington, DC.

Goodman, L. 1960. On the Exact Variance of Products. *Journal of the American Statistical Association* 55(4): 708–713.

Hemel, E. 1990. Fannie Mae and Freddie Mac: How Large are the Risks of Higher Capital Requirements? First Boston Equity Research: Boston, MA.

Hendershott, P.H. and R. Van Order. 1987. Pricing Mortgages: An Interpretation of the Model and Results. *Journal of Financial Services Research* 1(1): 77–111.

Housing and Community Development Act of 1992, Public Law102-550. Title XIII, Government Sponsored Enterprises.

HUD. 1992. *Report to Congress on the Federal Home Loan Mortgage Corporation. Information Statement*. Federal Home Loan Mortgage Corporation. March 30, 1990: 3, 5 and 30.

———. April 9, 1993.

Kau, J.B. and D.C. Keenan. 1994. An Overview of the Option-Theoretic Pricing of Mortgages. *Journal of Housing Research* 6(2): 217–244.

Marcus, A.J. and I. Shaked. 1984. The Valuation of FDIC Insurance Using Option-Pricing Estimates. *Journal of Money Credit and Banking* 16(4): 446–460.

Merton, R.C. 1987. An Analytical Derivation of the Cost of Deposit Insurance and Loan Guarantees. *Journal of Banking and Finance* 1: 3–11.

Omnibus Budget Reconciliation Act of 1990, Public Law 101-508. November 5, 1990.

Quigley, J.M. and R. Van Order. 1991. Defaults on Mortgage Obligations and Capital Requirements for U.S. Savings Institutions. *Journal of Public Economics* 44(3): 353–369.

Ronn, E.I. and A.K. Verma. 1986. Pricing Risk-Adjusted Deposit Insurance: An Option-Based Model. *Journal of Finance* 41(4): 871–895.

Shilling, J.D. Rates of Return on Mortgage-Backed Securities and Option-Theoretic Models of Mortgage Pricing. *Journal of Housing Research* 6(2): 265–284.

Spahr, R.W. and E.L. Escolas. 1986. Mortgage Guaranty Insurance: A Unique Style of Insurance. *Journal of Risk and Insurance* 57(2): 308–319.

Spahr, R.W., M.A. Sunderman and C.D. Amalu. 1991. Corporate Bond Insurance: Feasibility and Insurer Risk Assessment. *Journal of Risk and Insurance* 58: 418–437.

Spahr, R.W. and M.A. Sunderman. 1992. The Effect of Prepayment Modeling in Pricing Mortgage-Backed Securities. *Journal of Housing Research* 3(2): 381–400.

Stanton, T.H. 1991. *A State of Risk*. Harper Business: New York, NY.

United States Code: Congressional and Administrative News, 91st Congress-Second Session, Volume 2. Laws and Legislative History. West Publishing: Washington, DC.

United States General Accounting Office. 1988. *Housing Finance: Agency Issuance of Real Estate Mortgage Investment Conduits*. Washington, DC.

———. 1990. *Government Sponsored Enterprises: The Government's Exposure to Risks*. Washington, DC

United States Treasury, *Report of the Secretary of the Treasury on Government Sponsored Enterprises*. May 31, 1990. Washington, DC

Vandell, K.D. 1995. How Ruthless is Mortgage Default? A Review and Synthesis of the Evidence. *Journal of Housing Research* 6(2): 245–264.